LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **MATHEMATICS**

FIRST SEMESTER – **APRIL 2013**

MT 1816/1811 - REAL ANALYSIS

Date : 27/04/2013 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

Answer all the questions. Each question carries 20 marks.

I.a)1) If f is a continuous function on [a,b], then prove that $f \in R(\alpha)$ on [a,b].

OR

a)2) Define step function and prove: If a < s < b, $f \in R(\alpha)$ on [a,b] and $\alpha(x) = I(x-s)$, the unit step function, then prove that $\int_a^b f d\alpha = f(s)$ (5)

b) 1) Let $f \in R(\alpha)$ on [a,b] and $m \le f \le M$. Suppose that Φ is continuous on [m,M]. Define h(x) = Φ (f(x)), $x \in [a,b]$ then prove that $h(x) \in R(\alpha)$ on [a,b].

b)2) State and prove the fundamental theorem of calculus with reference to Riemann-Stieltjes integrals. (10+5)

OR

c)1) Suppose f is bounded on [a,b], f has only finitely many points of discontinuity on [a,b] and α is continuous at every point at which f is discontinuous then prove that $f \in R(\alpha)$.

c)2) State and prove the theorem on change of variables. (10+5)

II.a) 1) Suppose $\{f_n\}$ is a sequence of functions defined on E and suppose $|f_n(x)| \le M_n$ $(x \in E, n = 1,2,3,....)$. Then prove that $\sum f_n$ converges uniformly on E if $\sum M_n$ converges.

OR

a)2) Let $\mathcal{C}(X)$ denote the set of all continuous, complex valued, bounded functions on X. prove that $\mathcal{C}(X)$ is a complete metric space. (5)

b)1) State and prove the Stone -Weierstrass theorem. (15)

OR

c)1) Prove that there exists real continuous functions on the real line which is nowhere differentiable.

c)2) Suppose K is compact and the following three conditions hold good.

(i) $\{f_n\}$ is a sequence of continuous functions on K

(ii) $\{f_n\}$ converges pointwise to a continuous function f on K, and

(iii) $f_n(x) \ge f_{n+1}(x),\ \textit{for all}\ x \in K, n = 1,2,3,...$ Then prove that $f_n \to f$ uniformly on K.

(5+10)

III.a)1) Applying Riemann-Lebesgue lemma, prove the following: If $f \in L(-\infty,+\infty)$ then when $\alpha \to +\infty$, $\int_{-\infty}^{+\infty} f(t)\frac{1-\cos\alpha t}{t}dt = \int_{0}^{\infty}\frac{f(t)-f(-t)}{t}dt$.)

OR

a)2) Write a short note on the contribution of any two mathematicians who had contributed to the analysis on Fourier Series. (5)

b)1) State and prove Fejer's theorem and state the consequences of Fejer's theorem.

b)2) State and prove Riesz – Fischer's theorem. (9+6)

OR

c)1) State Jordan's test and Dini's test for the convergence of a Fourier series at a particular point.

c)2) State and prove Riemann – Lebesgue Lemma. (6+9)

IV. a)1) Let Ω be the set of all invertible linear operators on R^n. If Ω is an open subset of $L(R^n)$ then the mapping $A \to A^{-1}$ is continuous on Ω

OR

a)2) If $A \in L(R^n, R^m)$ then prove that $\|A\| < \infty$ and A is a uniformly continuous mapping of R^n into R^m. (5)

b)1) State and prove Implicit function theorem. (15)

OR

c)1) Prove that a linear operator A on a finite-dimensional vector space X is one-to-one if and only if the range of A is all of X.

c)2) Suppose E is an open set in R^n, **f** maps E into R^m, **f** is differentiable at x_o in E, **g** maps an open set containing **f**(E) into R^k and **g** is differentiable at **f(x$_0$).** Then prove that the mapping **F** of E into R^k defined by **F(x) =g(f(x))** is differentiable at x_o and **F'(xo)= g'(f(xo)) f '(xo).** (6+9)

V) a)1) How the chain rule of derivative is derived?

OR

a)2) Briefly explain the application of real analysis to real world issues. (5)

b)1) Explain the rate of change of a function with an illustration.

b)2) What is the area under a parabola bounded by a straight line segment? (7+8)

OR

c)1) Derive the expression for D' Alembert's wave equation for a vibrating string. (15)

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